GCSE Maths - Algebra

Nth term

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of nth term questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the nth term of the sequence 7, 10, 13, 16, 19, …

Step 1: Find the term-by-term difference.

For the sequence 7, 10, 13, 16, 19, … the term-by-term difference is 3.

Step 2: Use the term-by-term difference to make an estimate for the nth term.

If the nth term was $3n$ then the sequence would be 3, 6, 9, 12, 15, …

Step 3: Compare the estimate and work out the difference between the sequences generated. Use the difference to find the formula for the nth term.

n	**1**	**2**	**3**	**4**	**5**
3n	3	6	9	12	15
3n+4	**7**	**10**	**13**	**16**	**19**

Comparing the original sequence with the sequence with nth term $3n$, the difference between each term is $+4$. So, the nth term of the required sequence is $3n + 4$.

Guided Example

Find the nth term of the sequence 2, 9, 16, 23, 30, …

Step 1: Find the term-by-term difference.

Step 2: Use the term-by-term difference to make an estimate for the nth term.

Step 3: Compare the estimate and work out the difference between the sequences generated. Use the difference to find the formula for the nth term:

n	**1**	**2**	**3**	**4**	**5**

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Find the nth term of the following sequences:

a) 0, 4, 8, 12, 16, …

b) -5, -3, -1, 1, 3, …

c) -1, -5, -9, -13, -17, …

d) 0, -13, -26, -39, -52, ...

e) 1.1, 2.6, 4.1, 5.6, 7.1, …

Section B

Worked Example

For the nth term $4n + 1$, give the first 5 terms and the 90th term of the sequence

Step 1: Find the first five terms by substituting $n = 1, 2, 3, 4, 5$ into the nth term formula.

$$For\ n = 1: 4(1) + 1 = 5$$

$$For\ n = 2: 4(2) + 1 = 9$$

$$For\ n = 3: 4(3) + 1 = 13$$

$$For\ n = 4: 4(4) + 1 = 17$$

$$For\ n = 5: 4(5) + 1 = 21$$

*So, the first 5 terms are **5, 9, 13, 17, 21**.*

Step 2: Find the 90th term by substituting $n = 90$ into the nth term formula.

$$For\ n = 90: 4(90) + 1 = 361.$$

*So, the 90th term is **361**.*

Guided Example

For the nth term $3n - 8$, give the first 5 terms and the 90th term of the sequence

Step 1: Find the first five terms by substituting $n = 1, 2, 3, 4, 5$ into the nth term formula.

Step 2: Find the 90th term by substituting $n = 90$ into the nth term formula.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. For each of the following nth terms, give the first 5 terms and the 90th term of the sequence:

a) $-6n + 4$

b) $42n$

c) $13n - 1.5$

d) $-9n - 2$

Section C

Worked Example

Calculate the difference between the 13th term and the 25th term of the following sequence: 2, 6, 10, 14, 18, …

Step 1: Find the nth term of the given sequence.

For the sequence 2, 6, 10, 14, 18, … the term-by-term difference is $+4$*. If the nth term was* $4n$ *then the sequence would be 4, 8, 12, 16, 20, ...*

Comparing the original sequence with the sequence with nth term $4n$*, we see that the difference between each term is* -2*. So, the nth term of the required sequence is* $\boldsymbol{4n-2}$*.*

n	1	2	3	4	5
4n	4	8	12	16	20
4n-2	**2**	**6**	**10**	**14**	**18**

Step 2: Find the 13th and 25th terms.

$$\text{For } n = 13\text{: } 4(13) - 2 = 50$$
$$\text{For } n = 25\text{: } 4(25) - 2 = 98$$

Step 3: Find the difference between the two.

The difference between the 13th and 25th term is $98 - 50 = \boldsymbol{48}$*.*

Guided Example

Calculate the difference between the 19th term and the 61st term of the following sequence: -4, -7, -10, -13, -16, …

Step 1: Find the nth term of the given sequence.

n	1	2	3	4	5

Step 2: Find the 19th and 61st terms.

Step 3: Find the difference between the two terms.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Calculate the difference between the 82nd term and the 131st term of the following sequence: 2, -3, -8, -13, -18, …

4. Calculate the difference between the 49th term and the 89th term of the following sequence: -3, 6, 15, 24, 33, …

5. Calculate the difference between the 23rd term and the 91st term of the following sequence: 5.5, 8, 10.5, 13, 15.5, …

Section D

Worked Example

Is 194 a term in the sequence 7, 13, 19, 25, 31, … ?

Step 1: Find the nth term of the given sequence.

For the sequence 7, 13, 19, 25, 31, … the term-by-term difference is $+6$. If the nth term was $6n$ then the sequence would be 6, 12, 18, 24, 30, …

Comparing the original sequence with the sequence with nth term $6n$, we see that the difference between each term is $+1$. So, the nth term of the required sequence is $\boldsymbol{6n+1}$.

n	1	2	3	4	5
6n	6	12	18	24	30
6n+1	**7**	**13**	**19**	**25**	**31**

Step 2: Determine if the given term belongs to the sequence with the above nth term.

194 is a term in the above sequence if there exists a positive integer n where $6n+1=194$. So, we need to solve this equation:

$$6n+1=194$$
$$6n=193$$
$$n=\frac{193}{6}=32.1666\ldots$$

*Since n is not an integer, 194 is **not** a term in the given sequence.*

Guided Example

Is -2361 a term in the sequence -1, -3, -5, -7, -9, … ?

Step 1: Find the nth term of the given sequence.

n	1	2	3	4	5

Step 2: Determine if the given term belongs to the sequence with the above nth term.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

6. Is 925 a term in the sequence -5, -1, 3, 7, 11, … ?

7. Is 1389 a term in the sequence 13, 29, 45, 61, 77, … ?

8. Is -313 a term in the sequence 10, 8, 6, 4, 2, … ?

Section E - Higher only

Worked Example 1

Find the nth term of the following quadratic sequence 3, 9, 19, 33, 51, ….

Step 1: Work out the differences between the terms. Write the differences so that they form a new linear sequence.

3,	***9,***	***19,***	***33,***	***51, ...***
	+6	*+10*	*+14*	*+18*

Sequence of differences: 6, 10, 14, 18, ...

Step 2: Use the term-by-term rule of the sequence of differences to find the coefficient of n^2.

In the sequence of differences 6, 10, 14, 18, ..., the term-by-term rule is $+4$.

Since the original sequence is a quadratic sequence, it will have an n^2 *term in the formula. The coefficient of* n^2 *is always half of the term-by-term rule of the sequence of differences. In this case, the term-by-term rule is* $+4$ *so the coefficient of* n^2 *will be* 2.

Coefficient of n^2*:* 2

Step 3: Compare the given sequence with the quadratic sequence $\_\_n^2$ using the coefficient of n^2 found in **Step 2.**

$2n^2$	2	8	18	32	50
Sequence	**3**	**9**	**19**	**33**	**51**
Difference	+1	+1	+1	+1	+1

Step 4: Find the linear part of the quadratic nth term by finding the linear nth term of the new sequence of differences.

The new sequence of differences is 1, 1, 1, 1, 1, ...
So, the linear nth term for the sequence of differences is simply $+1$ *as each term in the sequence is the same.*

Step 5: Find the nth term of the quadratic sequence by combining the linear nth term of the sequence of differences found in **Step 4** and the coefficient of n^2 found in **Step 2**.

The linear nth term for the sequence of differences was $+1$ *and the coefficient of* n^2 *was found to be* 2*. So, the nth term for the quadratic sequence is*

$$\boldsymbol{2n^2 + 1}.$$

Worked Example 2

Find the nth term of the following quadratic sequence -17, -30, -49, -74, -105 ….

Step 1: Work out the differences between the terms. Write the differences so that they form a new linear sequence.

-17,		**-30,**		**-49,**		**-74,**		**-105, ...**
	-13		-19		-25		-31	

Sequence of differences: -13, -19, -25, -31, ...

Step 2: Use the term-by-term rule of the sequence of differences to find the coefficient of n^2.

In the sequence of differences, the term-by-term rule is -6.

Since it is a quadratic sequence, it will have an n^2 term in the formula. The coefficient of n^2 is always half of the term-by-term rule of the sequence of differences. In this case, the term-by-term rule is -6 so the coefficient of n^2 will be -3.

Coefficient of n^2: -3

Step 3: Compare the given sequence with the quadratic sequence $\_\_n^2$ using the coefficient of n^2 found in **Step 2**.

-3n²	*-3*	*-12*	*-27*	*-48*	*-75*
Sequence	***-17***	***-30***	***-49***	***-74***	***-105***
Difference	*-14*	*-18*	*-22*	*-26*	*-30*

Step 4: Find the linear part of the quadratic nth term by finding the linear nth term of the new sequence of differences.

For the new sequence of differences -14, -18, -22, -26, -30, … the term-by-term rule is -4. Comparing the sequence of differences with the sequence generated by the nth term $-4n$, there is a difference of -10 for each term. So, the nth term for the sequence of differences is $-4n - 10$.

n	***1***	***2***	***3***	***4***	***5***
-4n	*-4*	*-8*	*-12*	*-16*	*-20*
-4n-10	***-14***	***-18***	***-22***	***-26***	***-30***

Step 5: Find the nth term of the quadratic sequence by combining the linear nth term of the sequence of differences found in **Step 4** and the coefficient of n^2 found in **Step 2**.

The linear nth term for the sequence of differences was $-4n - 10$ and the coefficient of n^2 was found to be -3. So, the nth term for the quadratic sequence is

$$\boldsymbol{-3n^2 - 4n - 10}.$$

Guided Example

Find the nth term of the following quadratic sequence 5, 16, 33, 56, 85, ….

Step 1: Work out the differences between the terms. Write the differences so that they form a new linear sequence.

Step 2: Use the term-by-term rule of the sequence of differences to find the coefficient of n^2.

Step 3: Compare the given sequence with the quadratic sequence $\_\_n^2$ using the coefficient of n^2 found in **Step 2**.

__n²					
Sequence					
Difference					

Step 4: Find the linear part of the quadratic nth term by finding the linear nth term of the new sequence of differences.

Step 5: Find the nth term of the quadratic sequence by combining linear nth term of the sequence of differences found in **Step 4** and the coefficient of n^2 found in **Step 2**.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. Find the nth term of the following quadratic sequences

a) -10, -21, -40, -67, -102, ….

b) -2, 12, 34, 64, 102, …

Section F - Higher only

Worked Example

For the nth term $4n^2 + 1$, give the first 5 terms and the 83rd term of the sequence.

Step 1: Find the first five terms by substituting $n = 1, 2, 3, 4, 5$ into the nth term formula.

$$\textit{For } n = 1\text{: } 4(1)^2 + 1 = 5$$

$$\textit{For } n = 2\text{: } 4(2)^2 + 1 = 17$$

$$\textit{For } n = 3\text{: } 4(3)^2 + 1 = 37$$

$$\textit{For } n = 4\text{: } 4(4)^2 + 1 = 65$$

$$\textit{For } n = 5\text{: } 4(5)^2 + 1 = 101$$

*So, the first 5 terms are **5, 17, 37, 65, 101.***

Step 2: Find the 83rd term by substituting $n = 83$ into the nth term formula.

$$\textit{For } n = 83\text{: } 4(83)^2 + 1 = 27557.$$

*The 83rd term is **27557**.*

Guided Example

For the nth term $n^2 - 7$, give the first 5 terms and the 96th term of the sequence.

Step 1: Find the first five terms by substituting $n = 1, 2, 3, 4, 5$ into the nth term formula.

Step 2: Find the 96th term by substituting $n = 96$into the nth term formula.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

10. For each of the following nth terms, give the first 5 terms and the 123rd term of the sequence:

a) $-9n^2$

b) $n^2 - 3n + 5$

c) $-3.2n^2 + 9n + 5.1$

Section G - Higher only

Worked Example

A sequence has an nth term of $2n^2 + 4n - 10$.
Work out which term in the sequence has a value of 116.

Step 1: To find the value of n for which the given term corresponds to, set the nth term equal to the given term in the sequence.

$$2n^2 + 4n - 10 = 116$$

Step 2: Solve the quadratic equation.

$$2n^2 + 4n - 10 = 116$$
$$2n^2 + 4n - 126 = 0$$
$$n^2 + 2n - 63 = 0$$
$$(n - 7)(n + 9) = 0$$
$$n = 7 \text{ or } n = -9$$

Step 3: Select the value of n which is the non-negative integer. This n tells you which term in the sequence has the given value.

*Since n must be a non-negative integer, 116 is the term in the sequence corresponding to $n = 7$ and so 116 is the **7th term** in the sequence.*

Guided Example

A sequence has an nth term of $-4n^2 + 3$.
Work out which term in the sequence has a value of -61.

Step 1: To find the value of n for which the given term corresponds to, set the nth term equal to the given term in the sequence.

Step 2: Solve the quadratic equation.

Step 3: Select the value of n which is the non-negative integer. This n tells you which term in the sequence has the given value.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

11. A sequence has an nth term of $-5n^2 + 37n + 918$. Work out which term in the sequence has a value of 932.

12. A sequence has an nth term of $6n^2 - 64n - 150$. Work out which term in the sequence has a value of -128.